Limit state analysis of RC structures

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SUMMARY. A semi-analytical multi-scale approach is presented to introduce steel reinforcement into shear walls that allows one to carry out nonlinear analyses of full-scale reinforced concrete structures with reduced computational effort.

1 INTRODUCTION

The inelastic static pushover analysis has become a popular tool for evaluating the seismic capacity of structures. It is able of predicting the seismic force and deformation demands by accounting in an approximate manner for the inelastic redistribution of internal forces. Though approximate in nature and based on static loading, the pushover analysis can provide many significant insights into the structural behaviour and also put forward the design weaknesses that may be hidden in elastic analyses. The main features of the conventional pushover analysis are well described in [1], where are also emphasized limitations and possible causes that may produce loss of accuracy.

A basic prerequisite for successful applications of the method is an adequate knowledge of the inelastic behaviour of structural elements. This is particularly true for structures containing shear walls that, if not properly described, may significantly affect the results of the analysis. In this work we show how, under appropriate hypotheses, one can introduce steel reinforcement into shear walls by appealing to a semi-analytical multi-scale approach. In particular, reinforcements are taken into account using the usual conventional material behaviour, i.e. the the so-called parabolic-rectangular stress block for concrete and ideal elastic-plastic for steel as of Eurocode 2, and a fiber-free integration [2], that provides the exact solution for stress resultants over the cross section of a beam.

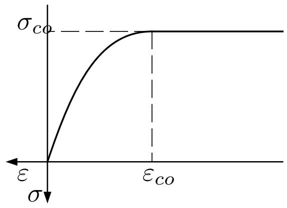
A representative numerical example is shown illustrating the capabilities of the proposed approach that, on one hand, allows one to carry out accurate nonlinear analyses of full-scale reinforced concrete structures with relatively reduced computational effort and, on the other side, prevents from meaningless results that can be arrived at when shear walls are modeled as beams.

2 MATERIALS

Assumptions and material properties adopted in the following are those generally used for the ultimate limit state analysis of RC structures. Our aim here is to avoid use of material parameters that may be unavailable in professional practice and make reference to building codes prescriptions, which require only basic material data.

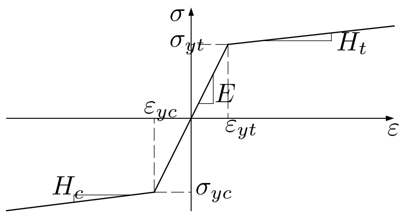
With this motivation, the nonlinear behavior of concrete is described by the usual parabolic-rectangular stress block, i.e.

$$\sigma_c(\varepsilon) = \begin{cases} 0 & \text{if } 0 < \varepsilon \\ \dfrac{\sigma_{co}}{\varepsilon_{co}}\left(2\,\varepsilon - \dfrac{\varepsilon^2}{\varepsilon_{co}}\right) & \text{if } \varepsilon_{co} \le \varepsilon \le 0 \\ \sigma_{co} & \text{if } \varepsilon < \varepsilon_{co} \end{cases}$$



where the tensile strength is neglected, σ_{co} is the peak compressive stress of concrete and ε_{co} is the corresponding strain. Reinforcement bars are assumed to behave according to a bilinear stress-strain relationship both in traction and compression:

$$\sigma_s(\varepsilon) = \begin{cases} H_c(\varepsilon - \varepsilon_{yc}) + \sigma_{yc} & \text{if } \varepsilon < \varepsilon_{yc} \\ E\,\varepsilon & \text{if } \varepsilon_{yc} \le \varepsilon \le \varepsilon_{yt} \\ H_t(\varepsilon - \varepsilon_{yt}) + \sigma_{yt} & \text{if } \varepsilon_{yt} < \varepsilon \end{cases}$$



where E is the elastic modulus, H is the hardening modulus, ε_y and $\sigma_y = E\varepsilon_y$ are the yield strain and stress while the subscripts c and t stand for compression and tensile, respectively.

The ultimate limit state is assumed to be attained when any of the two materials reaches a limit strain which is set to ε_{cu} for concrete in compression and to ε_{suc} and ε_{sut} for steel reinforcements.

3 CROSS SECTIONAL BEAM ANALYSIS

A Cartesian coordinate system with origin O and axes x and y lying on the plane of the cross section is introduced. Axis z is orthogonal to the plane $x - y$ and lies along the length of the beam. Each point of the section is defined by its in-plane position vector $\mathbf{r}$. Reinforcing bars are defined as concentrated areas A_{sj} of position $\mathbf{r}_{sj}$, $j = 1 \ldots n_s$, n_s being the number of re-bars. Euler-Bernoulli hypothesis and perfect bond between steel bars and concrete are assumed; strains in concrete and steel rebars are therefore provided by the same linear function ε given by:

$$\varepsilon(\mathbf{r}) = \epsilon + \mathbf{g} \cdot \mathbf{r} \tag{1}$$

ϵ being the axial strain at origin O and $\mathbf{g}$ the strain gradient. Stress resultants are evaluated by integrating the axial stress $\sigma_c(\varepsilon)$ over the concrete part Ω of the section to get the axial forces N and the bending moment vectors $\mathbf{M}^{\perp}$:

$$N_c = \int_\Omega \sigma_c[\varepsilon(\mathbf{r})] d\Omega; \quad \mathbf{M}_c^{\perp} = (-M_{cy}, M_{cx})^t = \int_\Omega \sigma_c[\varepsilon(\mathbf{r})]\mathbf{r} d\Omega \tag{2}$$

$$N_s = \sum_{j=1}^{n_b} \sigma_s[\varepsilon(\mathbf{r}_{sj})] d\Omega; \quad \mathbf{M}_s^{\perp} = (-M_{sy}, M_{sx})^t = \sum_{j=1}^{n_b} \sigma_s[\varepsilon(\mathbf{r}_{sj})]\mathbf{r} d\Omega \tag{3}$$

where subscripts c and s stand for concrete and steel, respectively. Stress resultants of the entire RC section are obtained from the superposition of the two above contributions.

4 RC SHEAR WALL

Main goal of this work is to show the capabilities of a shell element accounting for the presence of reinforcement bars. In particular, the 1D stress-strain relationships for steel and concrete described in the previous section are introduced in the direction of re-bars; this requires in turn to establish a relationship between strain and stress measures employed in beam and shells as well as that between elements dofs. The implemented shell element is a four-node flat quadrilateral obtaind by merging a plate bending element and a plane stress membrane. In particular, for the plate bending components a discrete Kirchhoff-based formulation is adopted; here the transverse shear energy is neglected altogether and the thin plate constraint is introduced in discrete form along the element edges to enforce the zero-shear strain condition [3]. As for the plane stress membrane, in-plane rotational degrees of freedom [4] are included in addition to the usual in-plane displacements; when combined with the plate bending part this provides a shell element possessing the 6 engineering degrees of freedom at the corner nodes, which allows to connect the shell with three-dimensional beam elements and prevents from singularity in planar configurations.

4.1 Numerical example

The numerical example concerns a planar symmetric RC structure consisting of a shear wall connected to two frames, see also Figure 1. The concrete material properties are $E_c = 70\,GPa$, $\nu = 0$ in the 1-direction (horizontal) while in the direction of the reinforcing bars (vertical) the parabola-rectangle stress block relationship is assumed with $\sigma_{co} = 40.0MPa$ and $\varepsilon_{c0} = 0.002$. The reinforcing steel constitution is ideal elastic-plastic with material properties $E = 210\,GPa$ and $\varepsilon_y = 0.002$. Reinforcements are made with 18 mm rebars that are uniformly distributed along the sides of the cross section with 20 cm spacing. Horizontal beams are subject to a uniformly distributed load of $20N/mm$ that is incremented up to a final value of $200N/mm$.

The shear wall is modeled either with the developed RC shell elements or using beam elements; in this last case rigid-end offset are added to the horizontal beams in order to account for the width of the shear wall. Figure 1 shows the deformed shapes and the limit states of the structure obtained in the two cases. The significant difference between the computed solutions is a direct consequence of the different kinematic models used to describe the shear wall. In particular, when using a beam element to represent the shear wall this last one is only subject to a normal force and does not experience stress concentrations. Basically, this occurs because of the symmetry of the geometrical model, that has the effect of rendering the horizontal beams perfectly buit-in on the symmetry axis; due to the rigid-end offsets the beam lengths and the bending moments are also reduced and no plastic hinge appears. On the contrary, when using shell elements for modelling the shear wall, the stress concentrations occurring in the vicinity of the wall-beam connections are correctly captured. This gives also rise to plastic hinges in this region which change the boundary conditions of the beams from built-in to simply supported. Accordingly, for increasing vertical load plastic hinges do appear also in the midspan region of beams that are responsible of the significant change of the deformed shapes and of the mechanisms depicted in Figure 1.

5 CONCLUSIONS

We have implemented a flat shell element for the analysis of RC structures containing shear walls. The proposed model makes use of 1D nonlinear constitutive laws for concrete; even with this limitation the benefits of the present formulation are clearly demonstrated by a numerical example that shows how neglecting the two-dimensional nature of a shear wall can lead to numerical results affected by gross inaccuracies.

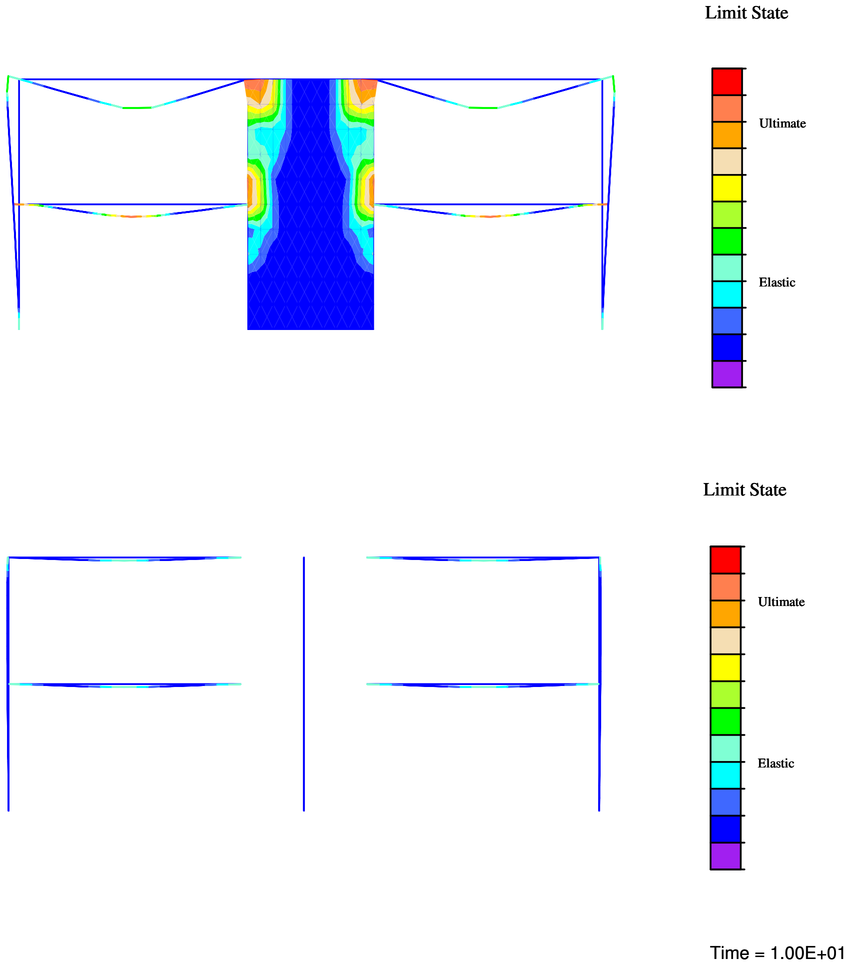


Figure 1: Mixed wall–frame structure. Deformed shape and Limit states.

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