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Robustness Analysis for Terminal Phases of Re-entry Flight

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I. Introduction

A novel approach to analyze the robustness of a Flight Control System (FCS) with respect to parametric uncertainties is presented, which specifically applies to gliding vehicles in the terminal phases of re-entry flight. Robustness analyses are particularly challenging for these systems. Their reference trajectories are appreciably time varying and encompass a broad variety of flight regimes. Furthermore, significant uncertainties on some critical design parameters affect the vehicle model, most notably those related to the aerodynamic behavior [1].

Current practice in FCS robustness analysis for this kind of application mainly relies on the theory of Linear Time Invariant (LTI) systems. In this approach, the original nonlinear system is linearized around a limited number of representative time-varying trajectories, including the nominal one. Then, the well-known frozen-time approach [2] is applied, yielding multiple LTI models. In this way classical stability margins [3], or more sophisticated LTI-based robustness criteria, such as μ-analysis [4] and D-stability analyses [5], can be evaluated. Recently, a Lyapunov based criterion coupled to Interval Analysis techniques[6] has been proposed for establishing robustness of a FCS. This approach does not resort to linearization of the system dynamics, but still requires the introduction of fictitious equilibrium points obtained by a frozen-time approach. Even if the flight experience demonstrated that frozen-time approaches are indeed operative, they are widely recognized as inefficient [7]. In fact, since the nominal trajectory may not be an equilibrium trajectory for the system in off-nominal conditions, frozen-time analyses can provide only indicative, and often heavily conservative, results.

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To overcome such problems, further investigations are usually performed to identify a limited set of worst-case combinations of uncertain parameters, to be used for FCS design refinement. In this case, nonlinear simulations in specific off-nominal conditions, selected using sensitivity analysis and designer's experience, represent the current practice. Optimization-based worst-case search has also been proposed [8], which may disclose the mutual effects of multiple uncertainties, but to a limited extent. In fact, the complexity of re-entry dynamics under multiple uncertainties implies that actual worst cases relevant for FCS design refinement are difficult to identify. In any case, worst-case analysis can select only a limited number of test cases, hiding possible further causes of requirement violations, so driving wrong refinement strategies that would not solve (or even worsen) FCS robustness problems.

Monte-Carlo (MC) analysis is, in practice, the only tool that is capable of investigating the combined effect of all uncertainties with a reasonable effort. However, being only a verification tool, when unsatisfactory robustness is discovered at this stage, the identification of its causes can require considerable post-processing effort [9]. This yields one of the major limitations of this approach, that is, the limited support to the FCS design refinement when a requirement violation occurs due to poor robustness. As a result, in these cases one is forced to iterate the design with scarce additional information.

The present paper contributes towards advancing the current practice used in robustness analysis for FCS design refinement, by introducing a method that takes into account nonlinear effects of multiple uncertainties over the whole trajectory, to be used before robustness is finally assessed with MC analysis. The method delivers feedback on the causes of requirement violation and adopts a robustness criteria directly linked to the original mission or system requirements, such as those employed in MC analyses. The first objective is achieved estimating the region of requirement compliance in the space of the uncertain parameters. In this way, the approach provides an exhaustive coverage of the uncertainty's effects on the FCL robustness. To translate mission requirements into robustness criteria over the whole trajectory, rather than at isolated points as in frozen-time approaches, we make use of the *practical stability* concept [10], which, to the authors' knowledge, has never been applied to robustness analyses of atmospheric re-entry vehicles.

II. Problem Statement

Let us assume to have a finite number p of parametric uncertainties, with zero nominal value, and ranging in a bounded set $\Pi \subseteq \Re^p$. The time-varying dynamical system representing the closed-loop augmented dynamics of an atmospheric re-entry vehicle can be written as in Eq.(1), where $f\colon [0,T]\times\Re^n\times\Pi \rightarrow \Re^n$ and $g\colon [0,T]\times\Re^n\times\Pi \rightarrow \Re^w$.

$$\dot{x} = f\left(t, x, \pi\right) \quad y = g\left(t, x, \pi\right) \tag{1}$$

We refer to time-varying nominal trajectories rather than stationary operating conditions, due to the possible lack of stationary equilibrium points for the dynamics of an un-powered re-entry vehicle in steep gliding flight. The common approach to determine robustness of system (1) is to rearrange the system dynamics in terms of variations with respect to the nominal trajectory, which becomes an equilibrium point. This allows using robustness criteria based on classical Lyapunov stability analysis. However, this approach cannot always be used for analyzing system robustness to uncertainties, since it is not guaranteed that the nominal trajectory is still an equilibrium trajectory in presence of non zero uncertainties. Such uncertainties, that not only cause perturbations in the dynamics but also modify the equilibrium trajectory, are known as *non-vanishing* [11].

The nonlinear robustness criterion proposed in the present work is based on the Practical Stability and/or Finite-Time Stability concepts [10, 12]. It requires only the inclusion of the system trajectories in a pre-specified time-varying subset of the state space, the admissible solutions tube $S_A(t)$, in face of bounded initial state displacements and disturbances. As opposed to the classical Lyapunov stability concept it does not require the existence of any equilibrium point, and is independent of Lyapunov stability, in the sense that one neither implies nor excludes the other. The practical stability criterion can deal with non-vanishing uncertainties and systems defined on a finite time domain. Moreover, it can handle robustness criteria directly linked to the original mission or system requirements, which are typically expressed in terms of trajectory tracking, thus identifying a $S_A(t)$ surrounding the trajectory to be tracked.

For simplicity, we do not consider deviations in the initial state, which is taken always equal to the nominal one, even though the proposed approach can include such deviations with minor modifications. The perturbed output trajectory $y(t;\pi)$ is thus defined as a trajectory of system (1) that starts at $t = 0$, $y(0) = y$□$_0$, under the constant input π. The robustness criterion is formulated as a Boolean property P depending on the uncertainties, being true when the criterion is satisfied.

$$P(\pi)=\begin{cases} true & y(t;\pi)\in S_A(t) \quad \forall t\in[0,T] \\ false & \exists t\in[0,T] : y(t;\pi)\notin S_A(t) \end{cases} \tag{2}$$

Willing to identify all the combinations of the uncertain parameters under which the system exhibits unsatisfactory robustness, the robustness analysis task is stated as determining the set $\Pi_A := \{\pi \in \Pi \mid P(\pi) = true\}$, consisting of all the uncertainties satisfying the robustness criterion. In this setting, the robustness analysis task can be re-formulated as a practical stability analysis problem, as follows.

Problem 1. Given system (1), a bounded set $\Pi \subseteq \Re^p$ such that $\pi \in \Pi$, a time-varying compact set $S_A(t)$ (admissible solutions tube), and the property P, determine the set Π_A.

III. Solution Approach

In order to simplify Problem 1 solution, let us assume the functions $f(\cdot)$,and $g(\cdot)$ to be differentiable in t, x and π over relevant domains, Π to be a p–dimensional hyper-rectangle, and the admissible solutions tube to be a w–dimensional hyper-rectangle for all $t \in [0,T]$. Various techniques exist that can deal with the practical stability analysis of a nonlinear dynamical system (see [12] for a survey), with the prominent approaches based on a Lyapunov-type analysis [10, 12]. Nevertheless, in spite of a wide range of literature on practical stability theoretical results, all the reported approaches have significant drawbacks when considered from an applicability perspective [13]. In this paper, an original approach is presented, which consists of two phases. First, the nonlinear vehicle dynamics are approximated within a pre-specified error tolerance by their time-varying linearizations in several off-nominal conditions (approximation phase). Then, Problem 1 is solved on the Linear Time Varying (LTV) systems obtained in the previous phase taking explicitly into account the approximation error (property clearance phase).

A. Approximation

Let us consider a partition $\{\Pi_k\}$ of the uncertainty domain, made of hyper-rectangular blocks Π_k, that is, a collection of subsets (blocks) that are both collectively exhaustive and mutually exclusive with respect to the set being partitioned. We then define a collection of LTV systems, each one approximating the nonlinear system in a single block. In particular, each LTV system is obtained by applying a first order expansion of $f(\cdot)$ and $g(\cdot)$ around x_k^0, π_k^0, where π_k^0 is the geometrical center of Π_k and x_k^0 is the state trajectory under π_k^0. In order to quantify the error made in approximating the nonlinear system with the LTV one, we use the weighted L_∞ norm distance between the nonlinear and linear trajectories. For each LTV system, and thus for each block Π_k of the partition, we define an approximation error function $e_k : \Pi_k \rightarrow [0,\infty[$ as:

$$e_k(\pi) := \left\| y(t;\pi) - y_{Lk}(t;\pi) \right\|_\infty^b \tag{3}$$

where $y_{Lk}(\cdot)$ stands for the trajectory of the LTV system defined in Π_k. We search for an approximation of the nonlinear system that introduces a pre-specified bounded error. Equivalently, this can be seen as searching for a partition $\{\Pi_k\}_L$ in which $e_k(\cdot)$ is below a pre-specified tolerance ε for all π in Π:

$$\{\Pi_k\}_L : \forall \Pi_k \in \{\Pi_k\}_L \quad \max_{\pi\in\Pi_k} e_k(\pi) \le \varepsilon \tag{4}$$

Differentiability of *f(·)* and *g(·)* functions assures that a partition complying to Eq. (4) may always be found, by using a partition of Π with sufficiently small blocks. Following this fact, $\{\Pi_k\}_L$ may be obtained by repeatedly shrinking the blocks of the partition for which the approximation error is higher than ε. The partition refinement is obtained iteratively, by means of an isotropic bisection technique, which splits a single p-dimensional hyper-rectangle "father" set in 2^p hyper-rectangular subsets. These "sons" are generated by bisecting each of the p one-dimensional intervals that define the father hyper-rectangle. At each iteration, the approximation error in each block Π_k is analyzed. Three cases are possible:

1) $\max_{\pi\in\Pi_k} e_k(\pi) \le \varepsilon$. The error is below the tolerance. Π_k is assigned to $\{\Pi_k\}_L : \{\Pi_k\}_L = \{\Pi_k\}_L \cup \Pi_k$.

2) $\max_{\pi\in\Pi_k} e_k(\pi) > \varepsilon$ and $\mathrm{vol}(\Pi_k) \le \eta$. The approximation error is higher than the tolerance and the volume of Π_k is smaller than a predefined maximum resolution η. In these blocks the system nonlinearities are so large as to prevent its LTV approximation within a small volume η and thus are not further considered for the subsequent step of the algorithm. Such blocks are left undetermined from the robustness analysis standpoint.

3) $\max_{\pi\in\Pi_k} e_k(\pi) > \varepsilon$ and $\mathrm{vol}(\Pi_k) > \eta$. Π_k is partitioned into 2^p sons and the process of evaluating the maximum approximation error is repeated for each of them.

1. Evaluation of Nonlinear Trajectories Approximation Error

Applying the previous algorithm requires checking that the distance between the nonlinear and linear trajectories under the same π is within the tolerance, for all $\pi \in \Pi_k$. Only a few approaches exist that allow relating the time responses of a nonlinear system to those of its linearization by quantitative means (e.g. [14–16]), either solving an optimization problem or providing bounds on the trajectory distance that are typically exponentially increasing with time, which limits their applicability. In order to tackle a wider class of problems, alternative approaches have been proposed in [17, 18], which estimate the approximation error introducing some heuristic methods.

In the present paper, we propose to evaluate the approximation error by probabilistic methods. In particular, by fictitiously introducing a statistical description of the uncertain parameters in the generic Π_k, we accept the risk of the approximation error being higher than the tolerance in a subset of Π_k having small probability measure. More precisely, we consider the nonlinear system to be well approximated in Π_k if the risk of $e_k(\cdot)$ being higher than the error tolerance is smaller than a threshold, which we take equal to 6%. By employing the well-known one sided Chebyshev inequality, the threshold on the risk of $e_k(\cdot)$ exceeding ε can be expressed in terms of $e_k(\cdot)$ mean and variance, yielding:

$$\Pi_k : \mathrm{E}(e_k) + 4\sqrt{\mathrm{Var}(e_k)} \le \varepsilon \;\Rightarrow\; \max_{\pi\in\Pi_k} e_k(\pi) \le \varepsilon \tag{5}$$

To determine the mean and variance of $e_k(\cdot)$, we use the Scaled Unscented Transformation (SUT) [19]. The SUT allows estimating the mean and covariance of the nonlinear function $e_k(\cdot)$ by propagating a set of deterministically chosen points through $e_k(\cdot)$ itself. These points are chosen based on the mean and variance of the independent variable: the uncertain parameters π in our case. We fictitiously assume π to be uniformly distributed in Π_k, and set up the SUT following the common practice in nonlinear Kalman Filtering applications (see [20] for more details).

B. Property Clearance

Once the $\{\Pi_k\}_L$ partition has been determined, one can obtain a solution to Problem 1 by formulating a similar problem on the LTV approximating systems corresponding to $\{\Pi_k\}_L$. For such LTV systems, the difference between any nonlinear and linear trajectories under the same π is included in a closed ball $B_\varepsilon \subset \Re^n$ with radius equal to ε. It follows that the nonlinear solutions tube is included in the Minkowski sum between the solution tube of its linearization and the former ball. To exploit this result in achieving the problem's solution, let us define a reduced admissible solution tube, obtained by shrinking $S_A(\cdot)$ of an amount equal to B_ε: $S'_A(t)$: $S'_A(t) \oplus B_\varepsilon = S_A(t)$, $\forall t \in [0,T]$. Consider now a modification of the P property, expressed in terms of $S'_A(\cdot)$ and of the linear trajectories corresponding to $\{\Pi_k\}_L$:

$$P'(\pi) := \begin{cases} true & y_{Lk}(t;\pi) \in S'_A(t) \quad \forall t \in [0,T] \\ false & \exists t \in [0,T] : y_{Lk}(t;\pi) \notin S'_A(t) \end{cases} \tag{6}$$

It can be easily proved that P' implies P. Therefore, introducing a region of admissible uncertainties analogous to Π_A, but based on P', as $\Pi'_A := \{ \pi \in \Pi \mid P'(\pi) = true \}$, it follows that $\Pi'_A \subseteq \Pi_A$.

1. *Computation of* Π'_A

Because of the definition of Π'_A and P', the determination of Π'_A may be seen as a set inversion problem. This can be solved by applying a set inversion algorithm, SIVIA (Set Inverter Via Interval Analysis), originally developed in the framework of Interval Analysis [21] and recently applied also to re-entry flight clearance [6]. The SIVIA algorithm allows one to compute an inner and an outer enclosure of Π'_A: $\underline{\Pi}'_A \subset \Pi'_A \subset \overline{\Pi}'_A$. The algorithm is iterative, and is initially applied to the partition $\{\Pi_k\}_L$. In order to determine if a block Π_k belongs to the enclosures, it performs an *inclusion test* $[P'](\Pi_k)$, being true (false) only if P' attains the same Boolean value over the whole block. More precisely, the inner enclosure $\underline{\Pi}'_A$ is composed of hyper-rectangular blocks Π_k for which the inclusion test is true. Because $\underline{\Pi}'_A \subset \overline{\Pi}'_A$, such blocks are also members of $\overline{\Pi}'_A$. Conversely, if it can be proved that $[P'](\Pi_k) = false$, then the block has an empty intersection with Π'_A, and it is thus rejected. Otherwise, no conclusion can be drawn based on the inclusion test, and the block Π_k is said undetermined. The latter is then bisected in 2^p subsets that are tested until their volume reaches the user-specified resolution η. Thus, such undetermined minimum-volume blocks are deemed small enough to be stored in the outer approximation $\overline{\Pi}'_A$ of Π'_A.

2. *Inclusion test for SIVIA*

The application of SIVIA requires defining an inclusion test, which is typically obtained by applying interval analysis, e.g. in [6]. However, interval computation is usually pessimistic, in the sense that a block Π_k may be deemed undetermined by an inclusion test even if the property under analysis attains the same Boolean value over the block itself. In the present context, we introduce a novel inclusion test that captures exactly the blocks in which P' is uniformly true, and also provide a condition that is sufficient for P' to be uniformly false.

The proposed inclusion test is based on a geometrical comparison of $S'_A(t)$ with the solutions tube corresponding to Π_k. The latter is determined exploiting the preservation of convexity in LTV trajectories under constant inputs. Let us consider a generic hyper-rectangular $\Pi_k \in \{\Pi_k\}_L$, which has 2^p vertices, $\pi_k^{(v)}$, by definition. Because the trajectory of an LTV system under a constant input π is an affine transformation with respect to π, any solution of the LTV system under a generic π in Π_k is a convex combination of the solutions under all the $\pi_k^{(v)}$. The knowledge of the 2^p vertex trajectories $y_{Lk}(t;\pi_k^{(v)})$ thus allows one to determine exactly the solutions tube corresponding to Π_k. By carrying out some algebra, here omitted for brevity, one can formulate an inclusion test that requires only a limited (and known a priori) number of linear trajectories, which are obtained by numeric simulation. Denoting as $[\ \ ]_i$ the i-th row of a matrix, and introducing the half-space representation of $S'_A(t)$ as $S'_A(t) = \{y \in \Re^w \mid S_A^L y \leq S_A^R(t)\}$, where $S_A^L = (I_{w\times w}, -I_{w\times w})^T$, $S_A^R : [0,T] \rightarrow \Re^{2w\times 1}$, yields the following inclusion test.

$$[P'](\Pi_k) := true \Leftrightarrow \forall t \in [0,T], \forall \nu = 1,..,2^p, \quad S_A^L y_{Lk}\left(t; \pi_k^{(\nu)}\right) \leq S_A^R(t) \tag{7a}$$

$$[P'](\Pi_k) := false \Leftarrow \exists t \in [0,T], \exists i = 1,..,2m : \forall \nu = 1,..,2^p \quad \left[S_A^L\right]_i y_{Lk}\left(t; \pi_k^{(\nu)}\right) > \left[S_A^R(t)\right]_i \tag{7b}$$

Applying the procedure discussed above, Π'_A is determined exactly within a prefixed resolution, and, due to the properties of the LTV systems defined on $\{\Pi_k\}_L$,, Problem 1 is solved conservatively for the nonlinear system (1). Nonetheless, the amount of conservativeness in estimating Π_A is bounded, and it can be reduced by decreasing the approximation error tolerance, at the price of a higher computational load.

IV. Robustness Analysis of the Longitudinal FTB1 Flight Control Law

The method is applied to evaluate the robustness of a candidate FCS for the longitudinal dynamics of the FTB1 vehicle, developed as part of the Unmanned Space Vehicle (USV) research and technology development program, managed by the Italian Aerospace Research Center (CIRA) [22]. The program aim is to develop and flight test key technologies in the disciplines of guidance, navigation and control, aerodynamics, and structures related to the terminal re-entry flight phase of a winged vehicle. The first flight test, on which the present paper focuses, is the first Dropped Transonic Flight Test (DTFT_1), carried out in February 2007 to investigate the transonic flight regime. The mission profile begins with a release from a stratospheric balloon at an altitude of 20 km, followed by a completely autonomous un-powered gliding flight, designed to reach the transonic regime at a constant angle-of-attack. The mission ends by deploying a parachute at a given subsonic Mach number, in order to safely splashdown in the Tyrrhenian Sea. The analyses concern the robustness against three uncertainties in the aerodynamic coefficients, which were determined to be the most influential by means of conventional sensitivity analyses [18].

A purely longitudinal nonlinear flight dynamics model is considered. The open-loop dynamics arise from well-known standard nonlinear longitudinal equations of motion. A detailed description of the FTB1 vehicle geometric and structural data can be found in [23]. According to the complete aerodynamic dataset, which is presented in [24], the lift, drag and pitching moment coefficients are given as the sum of a nominal and an uncertain aliquot. The former is predicted to be a nonlinear function of angle-of-attack α, Mach number M, altitude h, pitch-rate q and symmetric deflection of the elevons δ_e, which is the primary longitudinal control effector. Concerning the uncertain aliquot, we consider bias uncertainties in drag and pitching moment coefficients (C_{D0} and C_{m0}, respectively) along with the uncertainty in the effect of δ_e on the pitching moment coefficient ($C_{m\delta}$). The influence of uncertainties on the relevant aerodynamic coefficient is modeled by means of non-dimensional scaling functions $s(\cdot)$ that depend on the Mach number. The resulting aerodynamic coefficients functional dependencies are given in Eqs. (8), where the *nom* superscript denotes the nominal aerodynamic coefficient.

$$C_L = C_L^{nom}\left(\alpha, M, h, q, \delta_e\right) \tag{8a}$$

$$C_D = C_D^{nom}\left(\alpha, M, h, \delta_e\right) + s_{D0}\left(M\right)\cdot\pi_{D0} \tag{8b}$$

$$C_m = C_m^{nom}\left(\alpha, M, h, q, \delta_e\right) + s_{m0}\left(M\right)\cdot\pi_{m0} + \delta_e \cdot s_{m\delta}\left(M\right)\cdot\pi_{m\delta} \tag{8c}$$

After the first few seconds of the initial drop phase, a PID algorithm augments the open-loop vehicle dynamics. This is arranged in a cascade structure with feedback on pitch rate and angle-of-attack, with gains scheduled with respect to the dynamic pressure. The augmented dynamics are driven by a time-varying angle-of-attack command designed to fly a constant angle-of-attack of 7 deg. in the transonic region. With the model of Eq. (8) and the feedback action of the elevons, the longitudinal augmented vehicle dynamics take the form of Eq.(1) [20]. Three robustness criteria are enforced, based on mission requirements and FCS performance metrics. The FCS is required to track the reference angle-of-attack time-history with at most a 2 deg. error, as well as to avoid issuing commands that drive δ_e outside the range [-25, 25] deg. The Mach number is limited from above and below in the transonic region for complying to the mission objectives. These three robustness criteria naturally lend to a time-varying hyper-rectangular admissible solutions tube. The approximation phase has been performed allowing for a maximum distance between the nonlinear and linear trajectories of 0.27 deg. in α, $3\cdot10^{-3}$ in M, and 0.60 deg. in δ_e, to be achieved performing at most 5 bisections of the uncertainties domain, i.e. $\eta = 2^{-15}$. Numerical computation of the Jacobians for linearization of the nonlinear system in each Π_k is carried out every second. With this problem setting, a complete analysis requires ~12 minutes of execution time on a standard desktop PC equipped with a Pentium IV 2.4 GHz processor and 2GB RAM. Approximation phase results point out that the nonlinear system is successfully approximated in all Π within the allowed resolution. Figure 1 collects the property clearance results, in terms of the inner and outer enclosures of Π'_A.

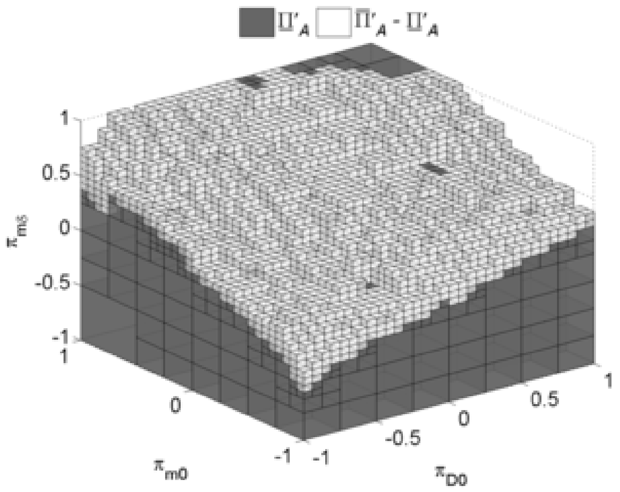

Fig. 1 Admissible region estimation

Validation of these results is performed by comparison with MC evaluation of the robustness criterion in Eq.(2). Since a similar behavior has been observed over the whole uncertainties' domain, data is shown only on a two dimensional slice of Π. Figure 2 compares a slice of the admissible region at constant $\pi_{m\delta} = 0.6$, with all samples of a MC evaluation in which the systems does not meet the robustness criterion. The method's ability of identifying the regions of unsatisfactory robustness is confirmed, as well as the predicted conservatism in the results.

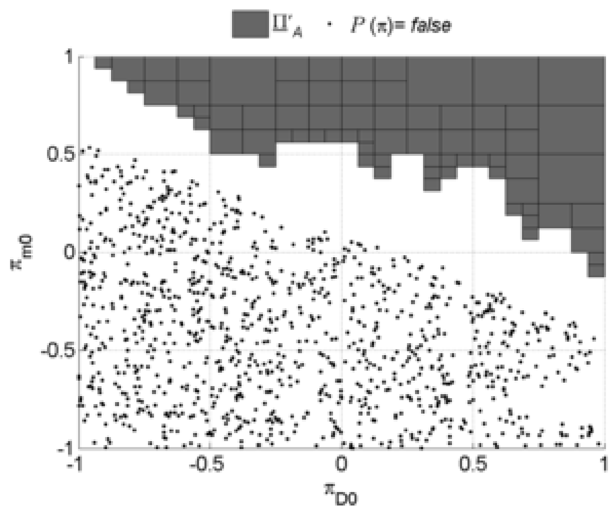

Fig. 2 Admissible region validation by MC analysis, $\pi_{m\delta} = 0.6$.

V. Conclusion

A novel approach to robustness analysis under parametric uncertainty has been presented. Its capability of highlighting the causes for requirement violations, being confident of having covered all possible combinations of the analyzed uncertain parameters, makes the developed technique an effective tool for driving the FCS refinement process. The practical stability property improves the accuracy in robustness evaluation with respect to frozen-time approaches, thus reducing the risk of discovering additional effects during robustness verification with Monte Carlo techniques. The number of uncertain parameters that can be simultaneously analyzed is the main concern of the method, due to the exponential increase in the computational load. Its application so far suggests that, when the method is executed on a standard desktop computer, the maximum dimension of manageable problems is on the order of five. A prior selection of the most influential uncertainties is thus necessary by using conventional sensitivity analyses.

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